

GEODYNAMICAL STUDIES USING GRAVIMETRY AND LEVELLING

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Academic dissertation in Geophysics

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Abstract

A consistent height system NH60 is created for the Baltic Sea area, using existing precise levellings. The mean sea surface topography of the Baltic Sea and its transition zone to the North Sea is determined in this height system from tide gauge records. The results agree with oceanographic models, according to which the major cause is the difference in density (salinity).

The change in gravity due to the Fennoscandian postglacial rebound is studied using relative measurements in Fennoscandia 1966–1993, and compared with elevation change. The results demonstrate that a mass compensation is taking place together with the rebound.

Vertical rebound rates in Finland are redetermined using three precision levellings. The properties of different estimators are studied and the sensitivity of the conclusions to various assumptions examined. No clear evidence of non-linearity in time is found but areas for further study are pointed out.

A tool for sensitivity analysis of sub-optimal estimation in the general linear model is developed. It is applied to the estimation of rebound rates from repeated precise levellings to show that no major differences can be expected between two traditional estimation techniques.

1. Introduction and list of papers

The subject of this thesis is the application of levelling and gravimetry to the study of geodynamical problems in the Fennoscandia postglacial rebound area and in the Baltic Sea. The thesis consists of six publications and this review. The publications will be referred to as Papers I to VI. They are

- I. Ekman, M. and J. Mäkinen: Mean sea surface topography in the Baltic Sea and its transition area to the North Sea: A geodetic solution and comparisons with oceanographic models. *J. Geophys. Res.* 101 (C5), 11993–11999 (1996).
- II. Ekman, M. and J. Mäkinen: Recent postglacial rebound, gravity change and mantle flow in Fennoscandia. *Geophys. J. Int.* 126, 229–234 (1998).
- III. Mäkinen J. and V. Saaranen: Determination of post-glacial land uplift from the three precise levellings in Finland. *J. Geodesy* 72, 516–529 (1998).
- IV. Mäkinen J. and V. Saaranen: Computation of post-glacial land uplift from the three precise levellings in Finland. Presented at the 13th General Meeting of the Nordic Geodetic Commission, Gävle, May 25–29, 1998. To appear in the Proceedings.
- V. Mäkinen, J.: Bounds for the difference between a linear unbiased estimate and the best linear unbiased estimate. To appear in *Phys. Chem. Earth A* 25, 693–698 (2000).

- VI. Mäkinen, J.: A bound for the Euclidean norm for the difference between a linear unbiased estimate and the best linear unbiased estimate. Accepted for publication in the *Journal of Geodesy* (2000).

In Paper I the consistent height system NH60 for the Nordic countries is defined and realized, using the national levelling networks as a basis. The height system is then used to derive mean sea surface topography at Finnish, Swedish, Danish, and Norwegian tide gauges in Baltic and its transition area to the North Sea.

Paper II deals with the relative gravity change derived from 27 years of observations on the Fennoscandian land uplift gravity line at 63°N, and with its relation to differences in postglacial rebound rates, observed with tide gauges and repeated precise levelling.

In Papers III and IV revised postglacial rebound rates for Finland approximately up to the latitude 66.5°N are obtained using three precise levellings

Papers V and VI originated in Paper III and a problem of sensitivity analysis: how to constrain the difference between (the numeric values of) estimators of rebound rates, when vaguely defined deviations from the computational model are allowed. Results valid for the general linear model are obtained, and applied to the rebound problem.

2. Contributions of the authors

Papers V and VI were conceived and written by me alone.

In Paper I the initiative was taken, and the first draft which only used Swedish tide gauges written by M. Ekman. I probably came upon the idea and the methods to join rigorously Swedish and Finnish national height systems. Norway and Denmark, and the oceanographic

connection across the Aland Sea were contributed by Ekman. I checked the Aland connection together with Finland and Sweden through an independent adjustment. Ekman wrote the paper, those parts which treat the Baltic were finalized by Ekman and me together.

I processed most of the gravimetric data in Paper II, and analyzed the land uplift difference in the Eastern part. The mass flow parameter, the single layer approximation, and the calculation of the remaining uplift are due to Ekman. I calculated the uncertainties and wrote the parts treating them. Ekman wrote the rest of the paper. The final version was then produced together.

For Paper III the levelling data were prepared by V. Saaranen. The software for computing the land uplift values was written and the calculations performed independently by both authors. MINQUE computations were done by Saaranen under my guidance. I performed the rest of the analyses and wrote the paper. The land uplift map, the network schematic, and the change in vertical velocities were drawn by Saaranen.

For Paper IV, Saaranen prepared the additional levelling data, and calculated and drafted the enlarged (compared with paper III) land uplift map and network schematic. I did the rest of the calculations and plots and wrote the paper.

3. Mean sea surface topography of the Baltic Sea

3.1 Principles

Determining mean sea surface topography (SST) along coasts with classical geodetic methods is straightforward: Use tide gauges to obtain the mean sea level (MSL) over some period of time, and connect these tide gauges with precise levelling. However, a number of questions require attention:

- (1) The levellings should be corrected for vertical motion such that the height differences refer to a common epoch.
- (2) The treatment of the permanent tide in the levellings should be such that the height differences are those of the mean crust over the mean geoid (Ekman, 1989).
- (3) The MSL at the tide gauges should be calculated such that they refer
 - (i) to a common epoch (which should be identical with that of the height differences) and
 - (ii) to a common period in time.

Usually (3)(i) is considered by fitting a regression line to the tide gauge record. Its ordinate at the given epoch is taken to represent the mean sea level. This takes care of linear trends, whether due to sea level (eustatic rise) or land motion (say, postglacial rebound). Further improvement is possible through (3)(ii), but in practice records cover different periods and contain gaps. In the Baltic, where sea level variation at the tide gauges is highly coherent, a reference gauge can be used to cor-

rect for this in the same way as in the determination of the trend (Ekman, 1996).

3.2 A century of determinations

Pioneering investigations

The first geodetic determinations of the topography of the Baltic were made by Blomqvist and Renqvist (1914), and Witting (1918), using the precise levelling networks of the surrounding countries and estimating the differences between their zero points.

In Finland the heights were in the NN-system based on the First Levelling of Finland (Blomqvist and Renqvist, 1910), in Sweden in the system later named RH00, based on the first high precision levelling of Sweden (Rosén, 1906). Although Blomqvist and Renqvist (1914), and Witting (1918) corrected the levellings for land uplift differences—estimated from tide gauges—the best they could, the calculated topography was in conflict with the oceanographic model of Witting (1918), due to levelling errors.

UELN-60 and related research

The next major geodetic calculation was based on the United European Levelling Network of 1960 or UELN-60 (Simonsen, 1960; Kääriäinen, 1960) where the epoch of heights was 1950.0. In Finland the Second Levelling was already complete in the area concerned (Kääriäinen 1960; 1966), but through Sweden only a single line of the second high precision levelling was available. In particular, none of the Swedish tide gauges south of Stockholm in the Baltic proper were connected.

The MSL heights were computed by Cahierre (1959). They refer to the epoch 1950.0. While special investigations (Rossiter, 1960) used the period 1940–1958, Cahierre (1959) applied as long records as possible. Slightly varying MSL heights (a couple of millimetres) are given on UELN-60 maps and in publications, depending on the phases A and B of the UELN-60, and on roundoff. The final numbers are in the Enclosure 1 of Simonsen (1960).

According to UELN-60 the slope of the Baltic SST (and of the transition area) is +21.9 cm from Smögen (Kattegat) to Stockholm, and +10.7 cm from Stockholm to Furuögrund (Bothnian Bay). On the Finnish coasts the slope from Hanko to Kemi (Bothnian Bay) is +5.1 cm, and from Hanko to Hamina (Gulf of Finland) +2.7 cm.

Bowden (1960) estimated the effect of density in Baltic Proper and in the transition area to the North Sea, and found that a slope of about +22 cm from Kattegat to the Aland islands could be expected, enough to explain the UELN-60 result from Smögen to Stockholm. On the other hand, Rossiter (1960; 1967) only included wind and pressure effects in his oceanographic model. Both were fitted empirically to observations using a

regression equation incorporating air pressure distribution (Doodson, 1960). The model explained nearly all of the UELN-60 slope around the Finnish coasts, but a residual of +18 cm from Smögen to Stockholm, and 7 cm from Stockholm to Furuögrund remained. In Sweden, Rossiter (1967) added stations not in the UELN-60, but their heights are apparently in RH00.

The oceanographic model of Lisitzin

The first extensive oceanographic model of the Baltic was developed in a series of papers by Lisitzin (1965; 1974), who included density, wind stress and atmospheric pressure. She found that its general slope from Kattegat to the Bothnian Bay was in agreement with the UELN-60.

Lisitzin (1966) also undertook a geodetic determination along the Finnish coasts in the height system N60 (Kääriäinen, 1966) which has epoch 1960.0. She used tide gauge records 1931–1960 and referred the sea level to 1960.5. Apart from the offset in zero, the geodetic results differ little from those of the UELN-60. Lisitzin (1966) noted their smaller slope in this area, compared with her model.

The second high precision levelling of Sweden becomes available

The Swedish network is essential to any determination of the Baltic SST using levellings. The second high precision levelling of Sweden was completed in 1967 and resulted in the height system RH70 (Anonymous, 1974), which has epoch 1970.0. Ussisoo (1977) determined the MSL in the epoch 1970.0 at Swedish tide gauges in the RH70 and found a slope +18.8 cm from Smögen to Stockholm, but only +2.8 cm from Stockholm to Furuögrund. The drastic reduction in the slope compared with UELN-60 turned out to be partly due to the treatment of the permanent tide: the RH70 heights refer to the non-tidal crust over the non-tidal geoid (Ekman, 1989).

Modern work

In the Unified European Levelling Network UELN-73/86 (Ehrensperger and Kok, 1986) the Swedish data is that of the second high precision levelling, in the non-tidal system. The Finnish data (which is that of the Second Levelling) and the Norwegian data, however, refer to the mean crust over the mean geoid, as is appropriate for oceanographic purposes. In the adjustment of the Nordic Block in the UELN-73/86 the tidal systems are confounded and it is not possible to correct it afterwards to either system.

Therefore Ekman and Mäkinen (1991) started from the national height systems of Sweden (RH70) and Finland (N60), which have epochs 1970.0 and 1960.0, respectively. The unified height system NH60, in the epoch 1960.0, in the mean tidal system and with the Normaal Amsterdams Peil (NAP) as zero was created.

The sea level heights were those given by Ussisoo (1977) and Lisitzin (1966), in the epochs 1970.0 and 1960.5, respectively. They were transferred to 1960.0.

The geodetic result of Ekman and Mäkinen (1991) gave a somewhat smaller MSL slope than the oceanographic model by Lisitzin (1974), especially in the Gulf of Bothnia and in the Gulf of Finland. Carlsson (1998) with her newer model later found that Lisitzin (1974) overestimated wind stress, and concluded that the main forcing for the slope is provided by horizontal density (salinity) and air pressure gradients.

In the meantime, Ekman (1994) extended the NH60 height system and the MSL investigation to Norway and Denmark in the transition area between the North Sea and the Baltic Sea. These results are incorporated in Paper I, together with an oceanographic connection over the Åland Sea. The geodetic results appear to be in excellent agreement with the oceanographic models (cf. Paper I).

Another recent geodetic determination along the Finnish coasts was made by Vermeer et al. (1988). They combined geodetic and oceanographic aspects by incorporating a barometric correction in the tide gauge readings. The results (in the epoch 1990.5) are given in the local reference only, not explicitly in a height system.

Comparison with other geodetic methods

While the agreement of the geodetic results of Paper I with recent oceanographic models is reassuring, in principle we cannot exclude the possibility that both are in error, in the same way as the levelling errors in the UELN-60 through Sweden were in agreement with the somewhat large slope in the model by Lisitzin (1974).

Three new geodetic determinations of the SST of the Baltic have recently become available and appear to corroborate the result of Paper I. The first (Kakkuri and Poutanen, 1997) is based on the Baltic Sea Level (BSL) GPS campaign of 1993 at tide gauges and on a modified BSL95A geoid by Vermeer (1995).

The GPS/geoid technique makes it possible to use tide gauges on islands, not connected to levelling networks. The SST has some anomalous features, but the general slope is the same as the one obtained in Paper I. However, the geoid was fitted to Finnish levelling which makes it dependent on the same levelling data as Paper I.

A later GPS/geoid determination at tide gauges by Poutanen and Kakkuri (2000) uses the BSL campaign of 1997 and the NKG96 geoid (Forsberg et al., 1997). The anomalous features have disappeared and the general slope is in agreement with Paper I. The same agreement is found in the altimetry/geoid SST of Poutanen and Kakkuri (2000) using the ERS-2, and again the NKG96 geoid.

While the NKG96 geoid was fitted to the Nordic levellings (Forsberg et al., 1997), and the altimetry to

the GPS/geoid SST (Poutanen and Kakkuri, 2000), in both cases the fit seems to be a single-parameter shift-of-level, and thus the SST slope obtained is independent of levelling and the result of Paper I.

Finally it should be mentioned that instead of starting with the national height systems and with the MSL computations of Lisitzin (1966) and Ussisoo (1977), and transforming them step by step, a completely different approach might have been adopted from the beginning:

- (1) Correct the Swedish data in UELN-73/86 to refer to the mean crust over the mean geoid.
- (2) Readjust the Nordic Block of the UELN-73/86.
- (3) Recompute the MSL from tide gauge observations to refer to 1960.0, and identical time spans.
- (4) Formalize the oceanographic tie of Paper I as a levelling line in the adjustment.

Such an independent computation has in fact been completed by the author in parallel with Paper I. This program does not constitute a genuine check of Paper I, as the observational data (levellings and tide gauges) are nearly identical, but it gives a check of the transformation procedures. Other advantages are: NH60 heights are obtained for all levelling points, accuracies can be estimated, and hypotheses formally tested. The results essentially confirm Paper I. They remain unpublished (Mäkinen and Ekman, 1999; Mäkinen, 2000).

4. Postglacial rebound, gravity change and elevation change

The oldest written documents on the contemporary emergence of land—or decrease of sea level—in Fennoscandia date from the 15th century, and the phenomenon has been the object of scientific curiosity at least since the 17th century. The following historical account is based on Kääriäinen (1953), Ekman (1991), and Kakkuri (1991).

Several hypotheses to explain the phenomenon were put forward, until the work of De Geer (1888, 1890) definitely established that indeed land was rising, due to the removal of the ice load at deglaciation. This idea had already been proposed by Jamieson (1865).

De Geer's observational material consisted of elevated shorelines. Soon stratigraphic methods (varved clays) were developed for dating them (Lidén, 1913). Such relative sea level histories from the Ångerman river (Sweden) and Oslo (Norway) were input to the rheological model of the uplift by Haskell (1935). Using a isoviscous incompressible half-space he obtained a mantle viscosity close to 10^{21} Pa s, a result which has stood the test of time remarkably well (Mitrovica, 1996). Some years later Niskanen (1948) already introduced a spherical model with lithosphere.

All the time, new observational data have been accumulated and put to use in Fennoscandia and North America: Trends in tide gauge records (Blomqvist and

Renqvist, 1914), tilts in lake levels (Gutenberg, 1933; Sirén, 1951), free air anomalies over the deglaciated areas (Vening Meinesz, 1937), repeated precise levelling (Kukkamäki 1939; Kääriäinen, 1953), repeated relative gravity measurements (Kiviniemi, 1974), polar wander (Nakiboglu and Lambeck, 1980), secular change in the Earth's low-degree zonal harmonics observed with satellite laser ranging (Rubincam, 1984), repeated absolute-gravity measurements (Lambert et al., 1994; Mäkinen, 1998; Jokela et al., 1999), motion measured with very long base line interferometry (James and Lambert, 1993) and with the GPS positioning system (Johansson et al., 1997; 2000; Mäkinen et al., 2000).

Starting with the pioneering work by Peltier (1974), the overwhelming majority of geodynamic models of the rebound apply a Maxwell linear viscoelastic rheology of the mantle. A Maxwell body can be represented as dashpot and spring in series. In the short time-range it reacts as an elastic body, in the long time limit as a Newtonian fluid. Typically, the purpose of the modelling is to constrain the viscosity profile of the mantle, or occasionally some other parameter of the Earth, say, lithosphere thickness. For most types of observables, an ice history is required input but may in turn be refined in the inversion process. For reviews see Wolf (1993) and Peltier (1998); for review of geodetic work in Fennoscandia see Kakkuri (1997).

In geodynamic modelling, the primary observations are the large amounts of relative sea level histories collected in different parts of the globe, nowadays predominantly dated with the calibrated radiocarbon method. Classical geodetic data (tide gauges and repeated levelling) which have yielded the major part of our knowledge about contemporary rates of uplift, have only quite recently been used to constrain the models (Lambeck et al., 1998; Davis et al., 1999).

Paper II is concerned with changes in gravity differences due to the rebound, papers III and IV with changes in elevation differences as determined with the three precise levellings in Finland.

4.1 Gravity change and postglacial rebound

The Nordic land uplift gravity lines

The thought of repeating gravity measurements for observing the postglacial rebound must have crossed the minds of many generations of geodesists until the increased accuracy made it a practical proposition. The late Professor R.A. Hirvonen, who in the 1930's performed a major part of the gravity measurements of the Finnish Geodetic Institute (Hirvonen, 1937) was four decades later fond of recounting an anecdote how he and his colleagues were contemplating the time span it would take for the gravity change to reach measurable proportions. The accuracy of their relative pendulum measurements was of the order of 1 mgal ($1 \text{ mgal} = 10^{-5} \text{ m s}^{-2}$).

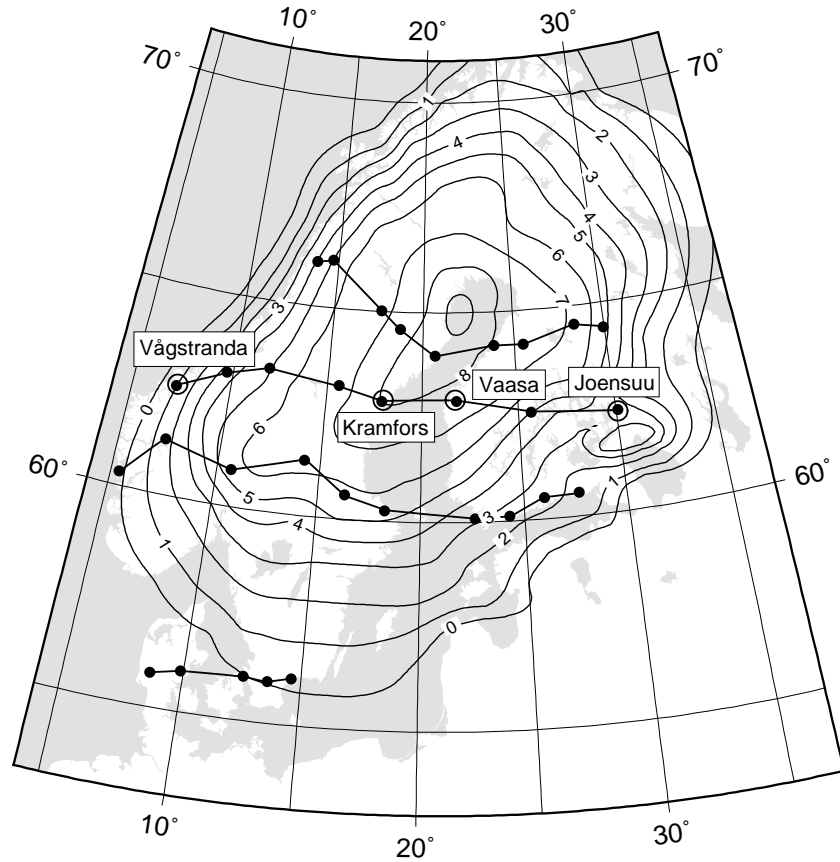


Figure 1. Land uplift relative to mean sea level (Kakkuri, 1997) as determined from tide gauges and repeated precise levellings, and the Fennoscandian land uplift gravity lines (Mäkinen et al., 1986). The named stations are used in this study. The isobases are generalized and the numeric values quoted in the text do not exactly correspond to them.

In 1963 with the Worden Master gravimeter, gravity differences could be measured with an accuracy of $30 \mu\text{gal}$ which was already sufficient to give the starting signal (Honkasalo and Kukkamäki, 1964). Measurements were begun in 1966 (Kiviniemi, 1974), with LaCoste & Romberg gravimeters and an estimated accuracy of $3\ldots 4 \mu\text{gal}$ per campaign, using several instruments in parallel. The accuracy is confirmed by the regression fits in Paper II.

Stations were chosen and constructed to have near-null gravity differences and maximum land uplift differences attainable in Finland, approximately along the latitude 63°N . Measurement methods were established, instrumental investigations performed, and a pattern of regular repeats set up by Kiviniemi (1974). In 1967 the line was extended westwards by Swedish and Norwegian geodesists (Pettersson, 1975). From this fundamental work we still benefit today. Later, three more lines along the latitudes 65°N , 61°N , and 56°N followed (Fig. 1).

Results of Paper II

To keep the discussion of the numeric values self-contained, several definitions and results from Paper II will be repeated here. Figure 1 shows the isobases of contemporary uplift relative to mean sea level, obtained with tide gauge records and repeated levellings. This apparent uplift rate \dot{H}_a is not, however, the whole story.

First, the mean sea level is undergoing a eustatic rise \dot{H}_e , about 1.2 mm/yr . Second, the rise of the mean sea level relative to equipotential surfaces like the geoid is not uniform: the sea surface topography is changing due to changes in salinity, winds etc. (Kakkuri, 1997). This part we neglect and write

$$\dot{H} = \dot{H}_a + \dot{H}_e \quad (4-1)$$

where \dot{H} is the uplift relative to the geoid. Finally, the geoid is changing at rate \dot{N} due to the postglacial re-

bound process itself. Summing these contributions we obtain the uplift rate relative to the Earth's centre of mass, often called the absolute uplift \dot{h}

$$\dot{h} = \dot{H} + \dot{N} \quad (4-2)$$

In terms of \dot{h} it is easy to construct simple geometric images of the uplift process. Two extreme models were pointed out by Honkasalo and Kukkamäki (1964). Suppose first that the uplift is caused by decompression with no mass added. Then at a point above the surface of the Earth and fixed relative to its centre of mass gravity is not changing at all

$$\dot{g} = 0 \quad (4-3)$$

At a point moving with the surface of the Earth, gravity change can be obtained using the free air gradient.

$$\dot{g} = -\frac{2g}{R}\dot{h} \quad (4-3)'$$

Suppose, second, that there is no decompression, but an inflow of mass in the upper mantle is pushing the crust upwards. At a point close above the Earth's surface and fixed relative to its centre of mass we have the Bouguer approximation

$$\dot{g} = 2\pi G\rho\dot{h} \quad (4-4)$$

and at a point moving with the surface

$$\dot{g} = -\frac{2g}{R}\dot{h} + 2\pi G\rho\dot{h} \quad (4-4)'$$

Here R is the radius of the Earth, G the Newtonian gravitational constant, and $\rho = 3300 \text{ kg m}^{-3}$ the density of the upper mantle. The use of the Bouguer approximation in (4-4) and (4-4)' for what at closer scrutiny turns out to be a stack of spherical lenses at different density interfaces might be questioned, but Wang and Kakkuri (2000) have shown that the error is negligible (Table 1, op. cit.). Similar considerations apply to (4-3) and (4-3)'.

From (4-3)' for the "free-air" model

$$(\dot{g}/\dot{h})_f = -\frac{2g}{R} = -0.31 \mu\text{gal mm}^{-1} \quad (4-5)$$

and from (4-4)' for the "Bouguer model"

$$\begin{aligned} (\dot{g}/\dot{h})_B &= -\frac{2g}{R} + 2\pi G\rho = -0.31 + 0.14 \\ &= -0.17 \mu\text{gal mm}^{-1} \end{aligned} \quad (4-6)$$

Thus observations of the ratio \dot{g}/\dot{h} can be used to discriminate between the two models.

The data of Paper II consists of observed differences in \dot{g} and \dot{H} within the western (W) and eastern (E) parts of the line 63°N (Fig 1). Writing symbolically with conventional one-sigma standard errors

$$(\dot{g}/\dot{H})_{obs} = \frac{-1.52 \pm 0.20 \mu\text{gal yr}^{-1}}{6.9 \pm 0.5 \text{ mm yr}^{-1}} \quad (\text{W}) \quad (4-7)$$

$$(\dot{g}/\dot{H})_{obs} = \frac{1.00 \pm 0.14 \mu\text{gal yr}^{-1}}{-4.7 \pm 0.5 \text{ mm yr}^{-1}} \quad (\text{E}) \quad (4-8)$$

Linearizing and using the number of degrees of freedom for the t -distributions in question we get the 95% confidence intervals

$$(\dot{g}/\dot{H})_{obs} = -0.220 \pm 0.086 \mu\text{gal mm}^{-1} \quad (95\%, \text{W}) \quad (4-9)$$

$$(\dot{g}/\dot{H})_{obs} = -0.213 \pm 0.080 \mu\text{gal mm}^{-1} \quad (95\%, \text{E}) \quad (4-10)$$

Then \dot{N} is calculated iteratively to obtain differences of $\dot{h} = \dot{H} + \dot{N}$ and

$$(\dot{g}/\dot{h})_{obs} = -0.208 \pm 0.086 \mu\text{gal mm}^{-1} \quad (95\%, \text{W}) \quad (4-11)$$

$$(\dot{g}/\dot{h})_{obs} = -0.200 \pm 0.080 \mu\text{gal mm}^{-1} \quad (95\%, \text{E}) \quad (4-12)$$

Approximate calculations of geoid change

At this point I will briefly discuss the computation of the geoid change rate \dot{N} . For the present purposes the geoid change rate \dot{N} appears only as a "correction", of the order of one part in ten, to be made to the \dot{H} values. In Paper II it is calculated for the land uplift maximum only, using a single layer density in the form of a cosine surface proportional to the velocity field \dot{h} (or \dot{H}). It comes out as 6 percent of \dot{H} at the maximum and for other points it is assumed to be roughly proportional.

Both assumptions are of course approximations: As to the error of putting all mass on the surface (rather than at different density interfaces), from Wang and Kakkuri (2000) one can deduct that at least for their stack of spherical lenses it is negligible (Table 1, op cit).

A number of authors (Dietrich, 1979; Sjöberg, 1982; Kakkuri, 1997) have converted a map of the apparent uplift \dot{H}_a to a grid of the time derivative of the free-air anomaly and integrated it with the Stokes kernel to obtain a grid of \dot{N} values. The calculated zero isobase of \dot{N} is then well beyond the zero isobase of \dot{H} used in the calculation, i.e., the two quantities certainly are not proportional for small values of \dot{H} .

However, a weakness in these calculations is that mass is not conserved, i.e., the integration stops at the zero isobase. Putting a negative belt beyond it would bring down all values of \dot{N} but especially those in the outskirts. The same remark applies to our model with the cosine surface and to the spherical caps of Wang and Kakkuri (2000).

The main result and discussion

I return to (4-11) and (4-12). Combining them

$$(\dot{g}/\dot{h})_{obs} = -0.204 \pm 0.058 \mu\text{gal mm}^{-1} \text{ (95\%)} \quad (4-13)$$

Thus the free-air model (4-5) is rejected, while the Bouguer model (4-6) is within the confidence interval.

There is plenty of other evidence against the free-air model: many signatures of postglacial rebound can only be explained if there is a change in gravity “as seen from space”, not only the change in gravity due to the movement of the observation point. One could at best maintain that while past rebound was associated with mass inflow, the present Fennoscandian uplift is due to some other mechanism (Mörner, 1991). But that alternative is rejected by our result.

At the other end of the range: on what grounds could one expect even a minor deviation from the Bouguer model, apart from speculating about some tectonic process superposed on the postglacial rebound? First, it should be remarked that the simple Bouguer model (4-4) turns out to be rather good an approximation for the viscous (i.e., mantle flow) contribution to gravity computed from geophysical models. Wahr et al. (1995) use a wide range of viscosity profiles and lithosphere thicknesses, and find that the viscous contribution is always about 1 μgal for 6.5 mm of uplift. This is 0.15 $\mu\text{gal mm}^{-1}$, and Wahr et al. (1995) show how it comes out of the model virtually as a Bouguer slab, with the PREM (Dziewonski and Anderson, 1981) densities they use.

In fact, based on the assumed relationship, repeated absolute gravity measurements are used in Greenland (Wahr et al., 2000) to eliminate the postglacial rebound contribution from observed vertical rates, to leave only the elastic contribution of variation in present-day glacier load. Similarly, in the Laurentide rebound area where contemporary vertical rates are not as well known than in Fennoscandia, they are inferred from repeated absolute measurements (Lambert et al., 2000).

Heuristically, mantle compressibility could be expected to modify this standard relationship between \dot{g} and \dot{h} . However, Wahr et al. (1995) included compressibility in the range of models that produced it. Moreover, Han and Wahr (1995) find explicitly that the relaxation modes associated with compressibility contribute negligibly to the vertical rates.

On the other hand, they point out that such conclusions depend on the properties (like bulk modulus) of the seismically deducted PREM model being valid on the much longer time scales of the postglacial rebound.

The question is also connected with the nature of the density discontinuities in the mantle at 420 and 670 km depth (material or isobaric, non-adiabatic or adiabatic), and phase changes at them (O’Connell, 1976; Christensen, 1985; Johnston et al., 1997). It would be highly useful to have predictions of the ratio \dot{g}/\dot{h} from such alternative models, to see whether

observed \dot{g}/\dot{h} could be used to constrain them. Unlike most observables, the datum \dot{g}/\dot{h} could also be expected to be relatively insensitive to details of ice history used in the prediction.

Finally, it should be remarked that the land uplift difference in the eastern part of line 63°N may require revision. In Paper II we used the result by Suutarinen (1983), based on the First and Second Levellings. Now, from three levellings (Paper III) the difference is larger by 0.3 mm yr⁻¹. Moreover, in paper II we used in Joensuu the closest surviving levelling bench mark of the First Levelling (in bedrock), no. 879 about 12 km northwest from the land uplift site.

However, there is another bench mark in bedrock, no. 64093 only 3 km south, which is from 1964 but has a local tie going back to a BM of the First Levelling. Using all three levellings (Paper III), the land uplift at no. 64093 is 0.4 mm yr⁻¹ smaller than at no. 879, a remarkable gradient over a distance of about 14 km.

The revised \dot{H} is

$$\dot{H} = -5.4 \pm 0.4 \text{ mm yr}^{-1} \quad (\text{E}) \quad (4-14)$$

where the (formal one-sigma) error estimate has 216 degrees of freedom instead of 9 in the denominator of (4-8).

The \dot{H} from (4-14) brings the initial value (4-10) down by 13 percent to

$$(\dot{g}/\dot{H})_{obs} = -0.185 \pm 0.059 \mu\text{gal mm}^{-1} \text{ (95\%, E)} \quad (4-15)$$

Observations (1996–1999) in the Finnish permanent GPS network FinnRef[®] lead to (Mäkinen et al., 2000)

$$\dot{h} = -5.3 \pm 0.4 \text{ mm yr}^{-1} \quad (\text{E}) \quad (4-16)$$

giving without iteration

$$(\dot{g}/\dot{h})_{obs} = -0.189 \pm 0.060 \mu\text{gal mm}^{-1} \text{ (95\%, E)} \quad (4-17)$$

To calculate (4-17), the error estimate in (4-16) was taken at face value (formal one-sigma at 152 degrees of freedom).

Thus the new uplift differences on the eastern side bring the result closer to the Bouguer model.

4.2 The determination of postglacial rebound from repeated precise levellings

Background

A review of methods and a case history of levellings in Fennoscandia can be found in (Vaníček et al., 1987) and in the references given there. Some main points are recounted here.

Papers III and IV use three precise levellings to investigate the postglacial rebound. It is not unique to have three levellings available for determining vertical motion. For instance, in Denmark the third precise levelling was completed in 1994 (Schmidt, 2000) and in

the Netherlands the fifth (!) primary levelling in 1999 (Molendijk et al., 2000). However, these countries belong to the fringe area of the Fennoscandian rebound, where rebound rates are small: around zero in Denmark and a minor subsidence in the Netherlands.

As to the regions with large rates vertical motion, much of the present-day Laurentide rebound is taking place in sparsely populated areas with hardly any levelling lines at all. In the rest of the rebound area in Canada and the northern United States, relevellings mostly consist of scattered (both in time and in space) line segments. The solution chosen by Carrera et al. (1991) was therefore to fit a smooth velocity surface to all the available primary velocity data: sea-level linear trends, tilt segments from lake level records, and tilt segments from relevellings.

In the central Fennoscandian rebound area, the situation varies from country to country. In Norway, the geography presents a number of obstacles to precise levelling. On some lines two or more levellings are available, but on others the question has not so much been how to use relevellings to obtain velocities, but how to obtain velocities to time-homogenize the levellings (Bakkeliid et al., 1996).

Sweden has been hampered by the somewhat sparse network of the first precision levelling (1886–1905). After the second precision levelling (1951–1967) was completed, Ussisoo (1977) determined rebound rates using jointly levellings and tide gauge data. In view of the precision of the two levellings ($4.4 \text{ mm}/\sqrt{\text{km}}$ and $1.6 \text{ mm}/\sqrt{\text{km}}$, respectively), the tide gauges dominate the results.

In Finland the precision ($1.3 \text{ mm}/\sqrt{\text{km}}$) of the First Levelling (1892–1910) and its good coverage up to latitude 65°N have been (and still are) a major asset. The first rebound results comparing First and the Second Levelling (1935–1955) were presented already by Kukkamäki (1939) for 4 loops in Southern Finland.

As the Second Levelling progressed, Kääriäinen (1953; 1966) determined land uplift differences for the whole area covered by the First Levelling. The precision ($0.64 \text{ mm}/\sqrt{\text{km}}$ for the Second Levelling) and the overlap of the networks made it possible to do this independently of tide gauge data and of any assumptions about spatial structure (say, a smooth velocity surface).

Three levellings give new possibilities

With the Third Levelling now available, it becomes possible to pose new questions:

- (1) How accurate the velocities (or more fundamentally, the levellings) "really" are?
- (2) Is there evidence of change in the velocities with time?

- (3) If yes, has it some structure, is it for instance related to velocities?

The questions (1) and (2) are inseparable. It is perhaps useful to interpret this using a simple regression analogy. Two levelling results on the same bench mark interval are two points in a scatter plot, with time as the abscissa. We can draw a straight line through two points with perfection. The only check we have for its validity is the closing error of the loop. I.e., the starting points of the N scatter plots belonging to the loop should sum nearly to zero, and the end point should do likewise. Thus the control is rather weak.

Suppose then that a third point is added on each plot. Now the straight line is controlled in each of them, not only in the sum. The proportion of redundant observations in the loop, which used to be $2/(2N)$ becomes $(N+3)/(3N)$. In the theory and terminology of Baarda (1968), the reliability of the observations and of the slope increase dramatically. This is independent of the improvement in precision that comes from the increased time span. (The precision *estimates* could even deteriorate if the fit becomes worse.)

But we may also ask whether the three-point fit is tight enough in view of the a-priori precision estimates based on the misclosures. If it is not, maybe these estimates are too optimistic and we should inflate them? How should we distribute this between the error bars of the three points? This could be important when we re-fit.

Alternatively, if we trust our a-priori error estimation, should we really draw a parabola (in Paper III we actually draw a polygon) through three points? Then we are again left with only the loop misclosures for control, with low redundancy and low reliability.

There is the added nuance of eliminating heights from the analysis. Since we are only interested in the velocities, why not subtract the levellings from each other to eliminate heights and work with velocities only? Would not this even bring an improvement if the levelled height differences on the same interval are correlated or share some systematic error?

However, the results turn out to be nearly the same, only the a-posteriori error estimation is affected. But the observed correlations raise another question: do the correlations express something physical, or are they fortuitous? The analogy can be continued to the elimination of the velocities to examine velocity changes only.

One may question whether our minute examination of the assumptions behind standard statistical procedures and of the "fine print" in the properties of classical estimators is worthwhile. If an argument really depends on them, would it not be best settled by employing completely different methods or different data? I think it is worthwhile, precisely in order to show where the data and/or the methods are inadequate. At least it is preferable to routinely calculating tail probabilities and taking them at face value.

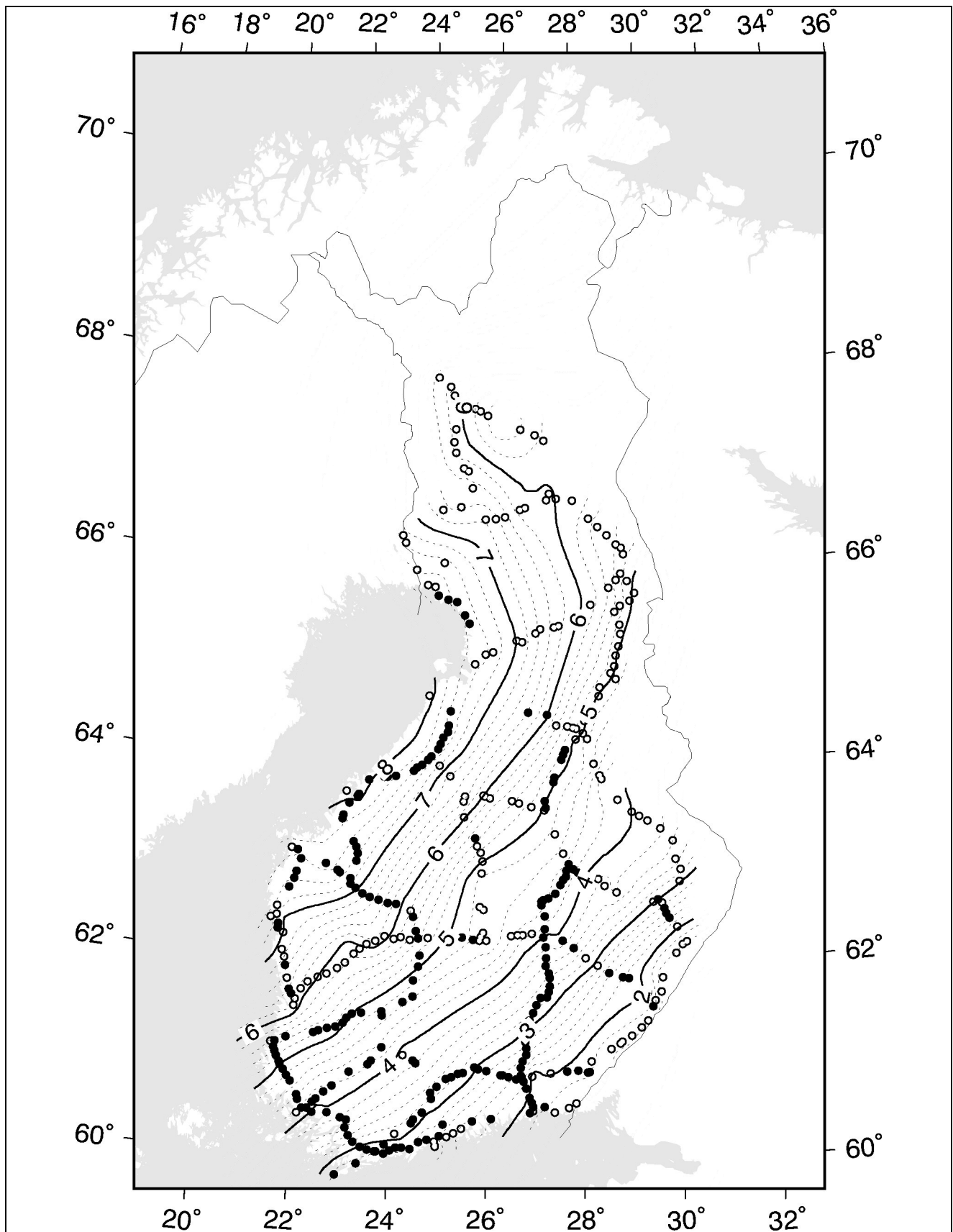


Figure 2. Land uplift relative to mean sea level, in mm/yr, from three precise levellings, network status 1999 (Mäkinen and Saaranen, 1999). Levellings were weighted according to the variance component estimation of Paper III. The tide gauge determination of Vermeer et al (1988) at Hanko (latitude 59°49', longitude 22°58') provides the starting value 2.73 mm/yr. The results of the map are not yet incorporated in general maps like that of Fig. 1.

Rebound from levelling in the GPS age

Nowadays highly accurate vertical rates can be obtained using permanent GPS stations. Four years of observations in the FinnRef[®] network yield the same precision (Mäkinen et al., 2000) between its 13 stations as the three levellings spanning nearly hundred years.

Nevertheless, GPS will not make rebound results from levellings obsolete. In view of the different spatial and temporal resolution and error characteristics of the two types of data they, for the moment, rather complement each other. For instance, it is not likely that FinnRef[®] style GPS data would be collected as densely as the levelling data we have, and on short distances repeated precise levelling still is better than campaign-style GPS.

Summary of findings

I finish this section by repeating some conclusions from Papers III and IV.

- 1) The intuitively attractive trick of forming differences between the levellings to eliminate the nuisance parameters (heights) and hopefully some unmodelled levelling errors from the determination of velocities hardly changes the estimated velocities but may have appreciable effects on their estimated uncertainties.
- 2) The effect on estimated uncertainties is due to correlations between epochs. In general, any deformation analysis is influenced by such correlations, independently of whether they are fortuitous or have physical foundation, and whether the formulation is univariate or explicitly multivariate. Even when the correlations are not high enough to be statistically significant themselves, they influence other conclusions.
- 3) There is an apparently significant change in velocities from the period between the First and the Second Levelling to the period between the Second and the Third Levelling. However, I do not trust this significance as it largely depends on a negative correlation between the First and the Third Levellings which appears spurious.
- 4) The observed (apparent) change in velocity is not related to observed velocity.
- 5) A-priori, given reasonable parameters for a rebound with exponential decay, there is no way its non-linearity could be detected by our levelling observations.
- 6) All apparent velocity change can be explained by levelling errors if we assume that the loop misclosures underestimate them by the factor 1/1.4.
- 7) This is true of the local anomalies detected by Lehmuskoski (1996), too. However, the fact that the distribution of crude velocity changes including these local anomalies is compatible with the normal distribution does not prove that they are pure levelling error. One way of investigating this might be to test rigorously whether their spatial distribution is

random, too, or whether correlation with some other indicators (say, ancient fault lines) could be detected.

- 8) Much more remains to be done with the data. For instance, the spatial distribution of the results has so far only been used to draw the plots.

The Third Levelling progresses further north every year. Figure 2 shows the land uplift map, status 1999 (Mäkinen and Saaranen, 1999).

5. Bounds for the difference between a linear unbiased estimate (LUE) and the best linear unbiased estimate (BLUE)

The impetus to write papers V and VI was given by an anonymous reviewer of Paper III. I was discussing the estimators (of velocities) which use the levellings (BLUE), and those which only use their differences (LUE). In the manuscript I wrote that "the small difference in their variances [the BLUE and the LUE] implies that the numeric values of the two estimators will be quite close for any reasonable observation vector". The reviewer was not convinced, and I realized my claim should be demonstrated.

It turned out that such a bound is an immediate consequence from first principles in Gauss-Markov theory. Assume our reference is the linear model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad \mathcal{E}\boldsymbol{\varepsilon} = \mathbf{0} \quad \mathcal{E}(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T) = \sigma^2\mathbf{G} \quad (5-1)$$

Take the simple case of a positive definite \mathbf{G} and a single (estimable) parametric function $\mathbf{u}^T\boldsymbol{\beta}$. If $\mathbf{d}^T\mathbf{y}$ is the BLUE and $\mathbf{c}^T\mathbf{y}$ a LUE for $\mathbf{u}^T\boldsymbol{\beta}$, and $\sigma^2\eta^2$ and $\sigma^2\kappa^2$ are their variances at the model (5-1), we have

$$|\mathbf{c}^T\mathbf{y} - \mathbf{d}^T\mathbf{y}| \leq (\kappa^2 - \eta^2)^{\frac{1}{2}} f^{\frac{1}{2}} \hat{\sigma} \quad (5-2)$$

for an arbitrary vector \mathbf{y} , i.e., not necessarily from model (5-1). Here $\hat{\sigma}^2$ is the a-posteriori variance factor from fitting the model (5-1) to the vector \mathbf{y} and f is the number of degrees of freedom in the model (5-1). Note that on the right-hand side of (5-2) we have separated $(\kappa^2 - \eta^2)^{\frac{1}{2}}$ which measures the sub-optimality of the LUE, and the fitting error $f^{\frac{1}{2}}\hat{\sigma}$ which characterizes the reasonableness of the observation vector \mathbf{y} . The right-hand side must depend on \mathbf{y} in some way, since \mathbf{y} on the left is an arbitrary vector.

I expected such a simple bound to be well-known, in view of the very large literature on the statistical consequences of mis-specification in the general linear model. However, a literature search revealed only three papers which applied the Euclidean norm to constrain the difference between the BLUE and a LUE. Haberman (1975) and Baksalary and Kala (1978) had treated the difference between the BLUE and the ordinary

least squares estimator (OLSE) of $\mathbf{X}\boldsymbol{\beta}$, a problem in simultaneous estimation of parametric functions but in other respects a special case. Baksalary and Kala (1980) had, in addition, allowed for a non-negative definite (n.n.d.) \mathbf{G} .

I then generalized (5-2) to cover a n.n.d. \mathbf{G} and the simultaneous estimation of parametric functions. The results of Baksalary and Kala (1978; 1980) were obtained as a special case. This is the subject of Paper VI.

Paper V applies the simple result (5-2) to the levelling problem in Paper III, which had originally motivated the research. Bounds are obtained for the differences “BLUE minus LUE” (estimators with and without heights) for all four combinations of the three levellings (three pairs and the full set).

However, Paper V has other aspects, too. Bounds for the BLUE minus LUE, similar to (5-2) but given in probability (i.e., confidence intervals) are generated using both the Chebyshev inequality and the normal distribution. The features in the structure of the levelling data which give rise to the difference between the BLUE and the LUE are identified.

In Paper V, bounds for the root-mean-square difference of the $q=199$ BLUEs and LUEs were generated by summing the q individual bounds. The results of Paper VI on simultaneous estimation of parametric

functions make it possible to sharpen these bounds considerably (Table 1).

6. Summary and conclusions

A consistent height system NH60 was created for the Baltic Sea area. The mean sea surface topography of the Baltic Sea and its transition zone to the North Sea was determined in this height system from tide gauge records. The results agree with oceanographic models, according to which the major cause is the difference in density (salinity).

The change in gravity due to postglacial rebound was studied using relative measurements in Fennoscandia 1966–1993, and compared with elevation change. The results demonstrate that a mass compensation is taking place together with the rebound.

Vertical rebound rates in Finland were redetermined using three precision levellings. No conclusive evidence of non-linearity in time was found but further study is warranted.

A tool for sensitivity analysis of estimation in the general linear model was developed. It was applied to the estimation of rebound rates from repeated precise levellings to show that no major differences can be expected between two traditional estimators.

Table 1. Computation of Euclidean bounds for the root-mean-square difference between the velocity estimators $\hat{\dot{h}}_i$ (LUE), which only use the differences between levellings, and the $\hat{\dot{h}}_i$ (BLUE), which use the levellings themselves, for the four datasets formed out of the three levellings in the common network (Paper V). Rows 1...4 are rows 1, 2, 3, and 12 of Table 3, Paper V. Row 6 gives a tighter bound than Row 4, calculated using Row 5 and the theory in Paper VI. Rows 5 and 6 are previously unpublished. The notation combines Papers V and VI.

Row	Quantity	Symbol	Unit	Dataset (i.e., levellings used)			
				1,2	2,3	1,3	1,2,3
1	Degrees of freedom	f		18	18	18	225
2	Sum of variances of $\hat{\dot{h}}_i$ (BLUE)	S_0^2	(mgpu/yr) ²	15.359	10.142	5.569	5.072
3	Sum of variances of $\hat{\dot{h}}_i$ (LUE)	S_1^2	(mgpu/yr) ²	15.829	10.325	5.583	5.126
4	Euclidean bound for $\left[\frac{1}{q} \sum_{i=1}^q \left \hat{\dot{h}}_i - \hat{\dot{h}}_i \right ^2 \right]^{\frac{1}{2}}$ at $\hat{\sigma} = 1$ using the trace of $(\mathbf{C}-\mathbf{D})^T \mathbf{G} (\mathbf{C}-\mathbf{D}) = \mathbf{C}^T \mathbf{G} \mathbf{C} - \mathbf{D}^T \mathbf{G} \mathbf{D}$	$\left[\frac{1}{q} (S_1^2 - S_0^2) \right]^{\frac{1}{2}} f^{\frac{1}{2}}$	mgpu/yr	0.206	0.129	0.036	0.247
5	Largest eigenvalue of $(\mathbf{C}-\mathbf{D})^T \mathbf{G} (\mathbf{C}-\mathbf{D}) = \mathbf{C}^T \mathbf{G} \mathbf{C} - \mathbf{D}^T \mathbf{G} \mathbf{D}$	θ_1		0.0731	0.0316	0.000244	0.00909
6	Euclidean bound for $\left[\frac{1}{q} \sum_{i=1}^q \left \hat{\dot{h}}_i - \hat{\dot{h}}_i \right ^2 \right]^{\frac{1}{2}}$ at $\hat{\sigma} = 1$ using the eigenvalue θ_1	$\frac{1}{q^{\frac{1}{2}}} \theta_1^{\frac{1}{2}} f^{\frac{1}{2}}$	mgpu/yr	0.0813	0.0534	0.00470	0.101

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