MAA-AMET
Estonian Land Board

Estonian gravity network in 2009

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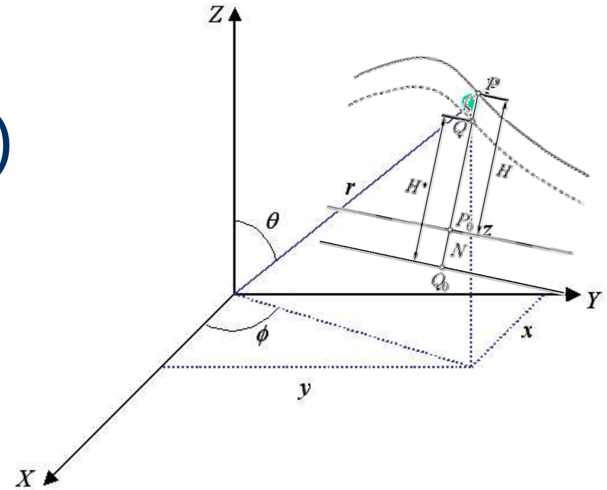
Reference system

- Estonian Geodetic System (since 2004 enforced by the Regulation of the Government):
  - geodetic reference system (*X*, *Y*, *Z*)
  - height system (*H*, *N*)
  - gravity system (*g*)



- Estonian gravity system (EGS):
  - based on the IAGBN* standards
  - realization is <u>network **GV-EST95**</u> (divided into I, II and III order)
  - epoch 1995.8
  - zero tidal system

*International Absolute Gravity Basestation Network

GV-EST95 – Instruments

- 2001-2004 LCR-G (Nr 4, 113, 115)
  - reso: 1-10 µGal
  - accuracy: 5…200 µGal
- 2003-... Scintrex CG-5 (Nr. 36, 10092)
  - reso: 1 µGal
  - accuracy: 2…?? µGal

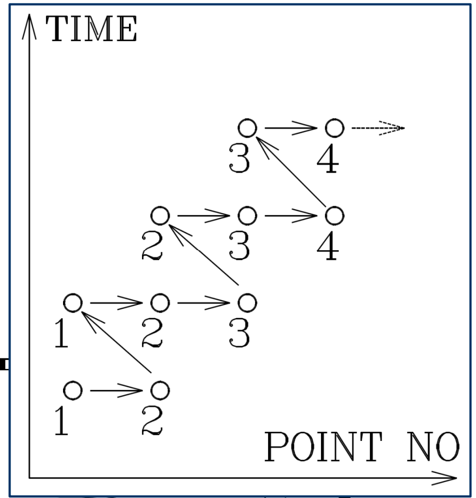
1 mGal = 1000 µGal = 10^{-5} m/s^2

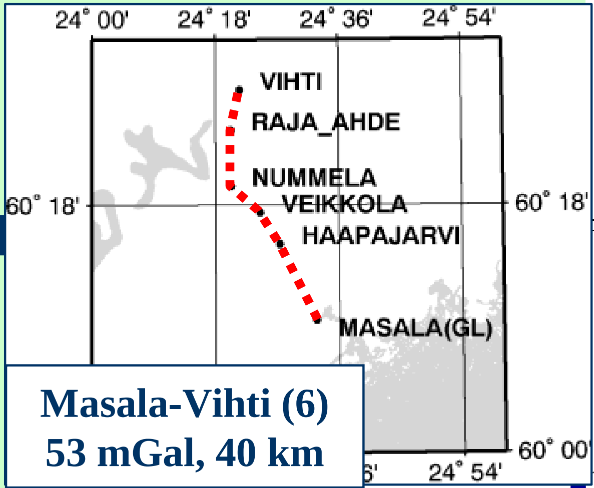
GV-EST95 – Observations

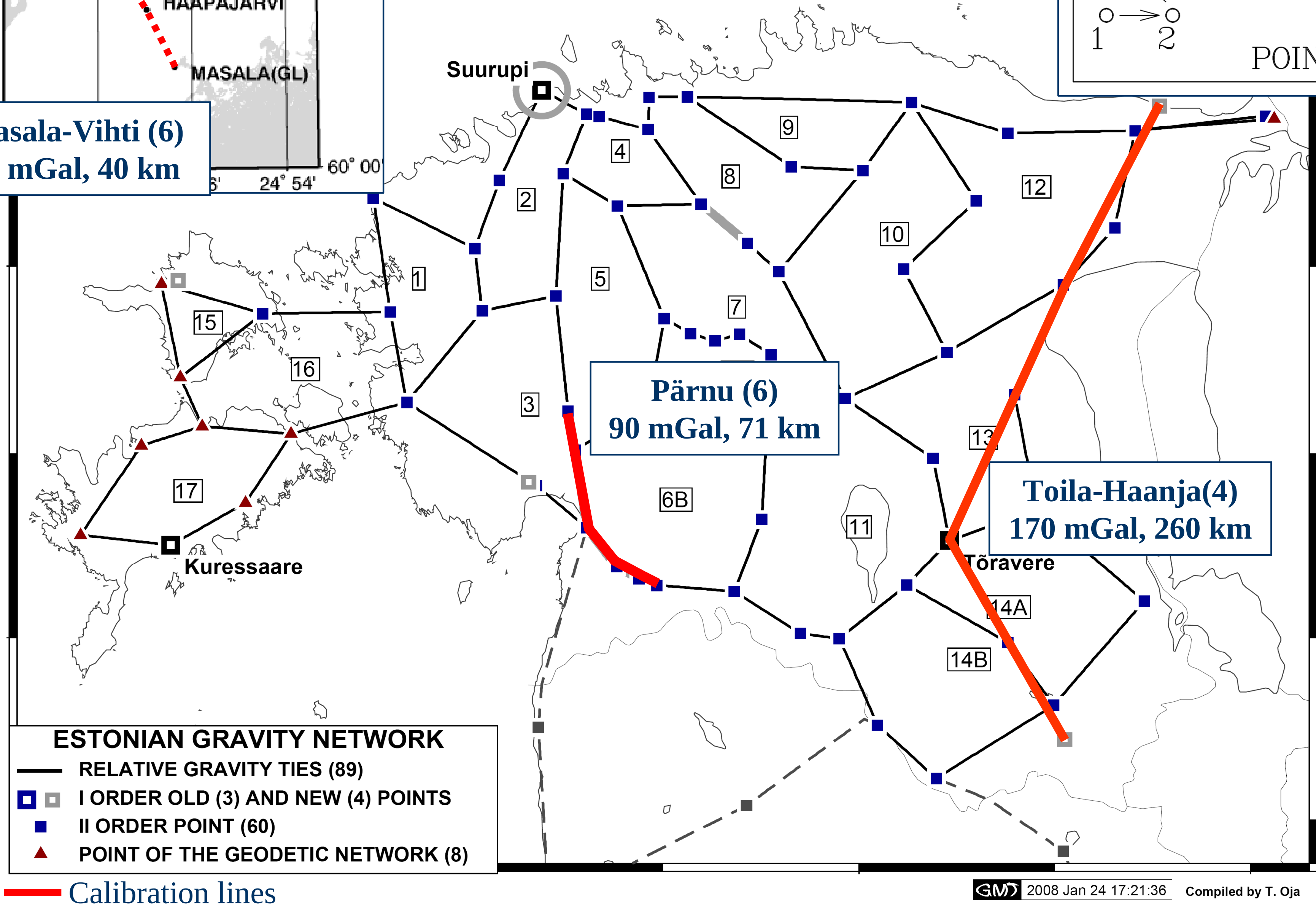
- In 2003 the whole network was observed with 3 LCR-G meters (totally 71 points and 89 ties)
- Additional measurements in 2001-2002 and 2004-…
- High precision observations along the calibration lines in Estonia and in Finland
- In 2006 high precision ties between Latvian and Estonian gravity network points (coop.with the LGIA*)
- 2007-2008 two absolute gravity campaigns by IfE** and FGI*** (alltogether 9 point obs.)

*Latvian Geospatial Information Agency
**Institut für Erdmessung, Leibniz Universität Hannover
***Finnish Geodetic Institute

– Observations







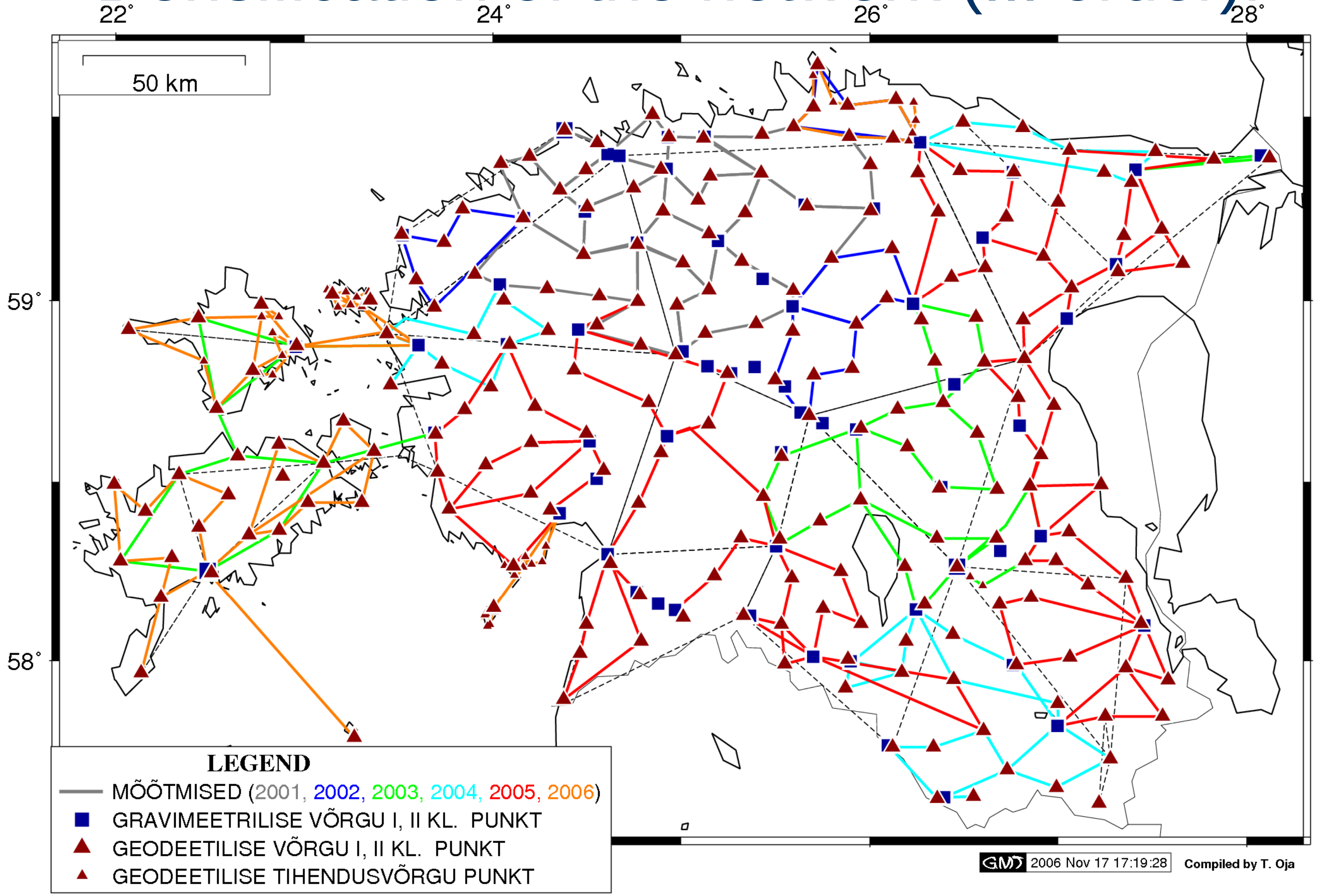
Calibration lines

GV-EST95 – Observations

Absolute measurements in Suurupi (SUUR): IfE team in 2007; FGI team in 2008

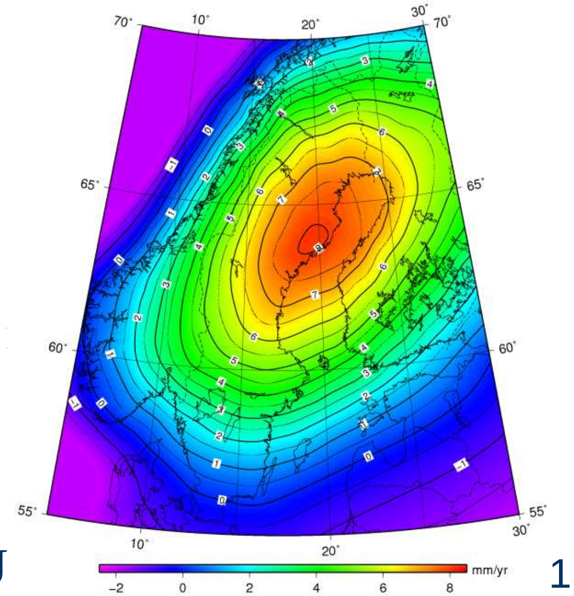
GV-EST95 – Observations

- Densification of the network (III order):



GV-EST95 – Data process

- **Calibration corrections**: linear and nonlinear parts
- **Tidal corrections**: Tamura (1987) tidal potential development, local parameters for the wavegroups from the global grid (Wenzel,Timmen 1994)
- **Atmospheric correction**: local air pressure, normal pressure (DIN 5450) and the coefficient –0.3 µGal/hPa
- **Sensor height reduction**:
  for LGR-G h_{red} ~ 5-10 cm,
  for CG5 h_{red} ~ 25 cm (11 cm)
- **PGR corrections** from the uplift model



NKG2005LU

GV-EST95 – Functional model

Previous observation model (program GRADJ):

$$y(t) = g + N_0 + Dt$$

New model (GRADJ2):

$$y(t) = g^{T_0} + \dot{g}^{T_0}(t - T_0) + N_0 + \sum_{p=1}^{r} D_p (t - t_0)^p + \sum_{k=1}^{m} Y_k z^k + \sum_{l=1}^{n} A_l \sin(2\pi z / P_l + \varphi_l)$$

Temporal change

Polynomial drift

Calibration

GV-EST95 – Network adjustment

- Optimal unbiased estimation for **x** by WLS:

$$\hat{\mathbf{x}} = (A^T W_{\mathbf{b}} A)^{-1} A^T W_{\mathbf{b}} \mathbf{b}$$

- variance of a single reading $\Rightarrow$ ***W***
- Covariance matrices:

$$\Sigma_{\hat{\mathbf{x}}} = \hat{\sigma}^2 \left(A^{\mathrm{T}} W_{\mathbf{b}} A \right)^{-1}, \quad \Sigma_{\hat{\mathbf{r}}} = \hat{\sigma}^2 \left(W_{\mathbf{b}}^{-1} - A \left(A^{\mathrm{T}} W_{\mathbf{b}} A \right)^{-1} A^{\mathrm{T}} \right)$$

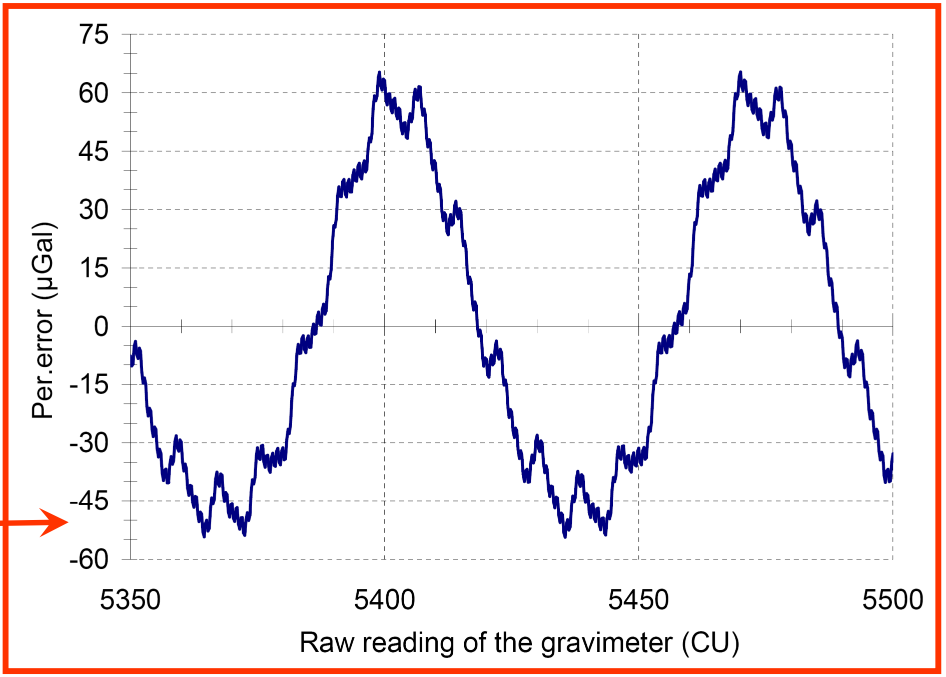
- Statistical tests: Student's *t*-test, χ^2 statistic, Pope's τ-test (at a 95% conf. level)

GV-EST95 – Results

- correctional calibration function for LCR-G gravimeters:

Gravimeter	G-4		G-113		G-115	
F_{Pol}, Y_1	0.996228 (±49)		1.000164 (±26)		1.000038 (±23)	
F_{Per} P (C.U.)	E	φ	E	φ	E	φ
1.0000	4.4±1.1	322±14	2.6±0.6	235±14	1.7±0.5	69±17
3.9412	- -	- -	2.3±0.6	196±13	- -	- -
7.8824	4.5±1.0	166±16	6.3±0.6	268 ±5	3.7±0.6	53 ±9
35.4706	6.0±1.2	79±14	5.8±0.7	7 ±6	11.1±0.6	212 ±3
70.9412	4.3±1.7	57±23	**52.5**±0.8	326 ±1	8.7±0.7	12 ±6

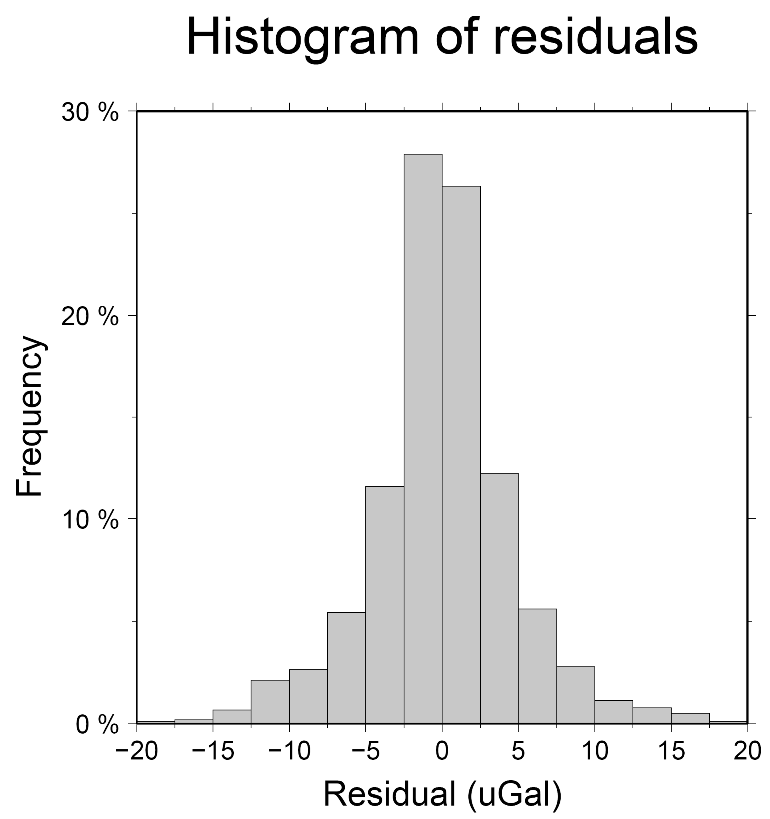
*Amplitudes (E) in μGal



- no significant calibration errors for Scintrex CG-5s (situation in 2006)

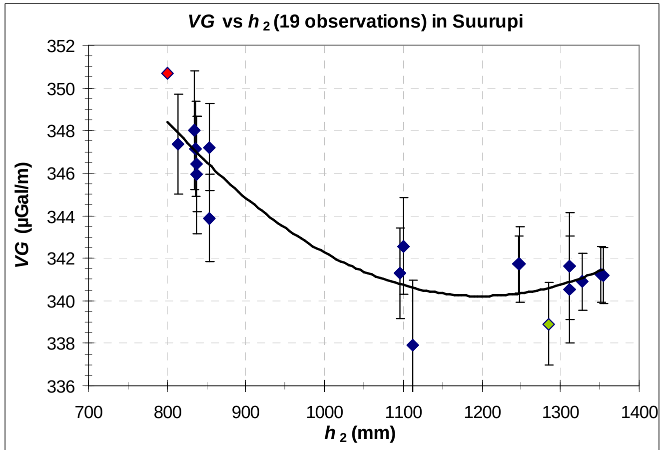
GV-EST95 – Results (preliminary)

- Adjustment with 3 fixed stations (2004-09-14)
- observed with the JILAg-5 of FGI in 1995
- RMS of the residuals ≤ ±10 µGal

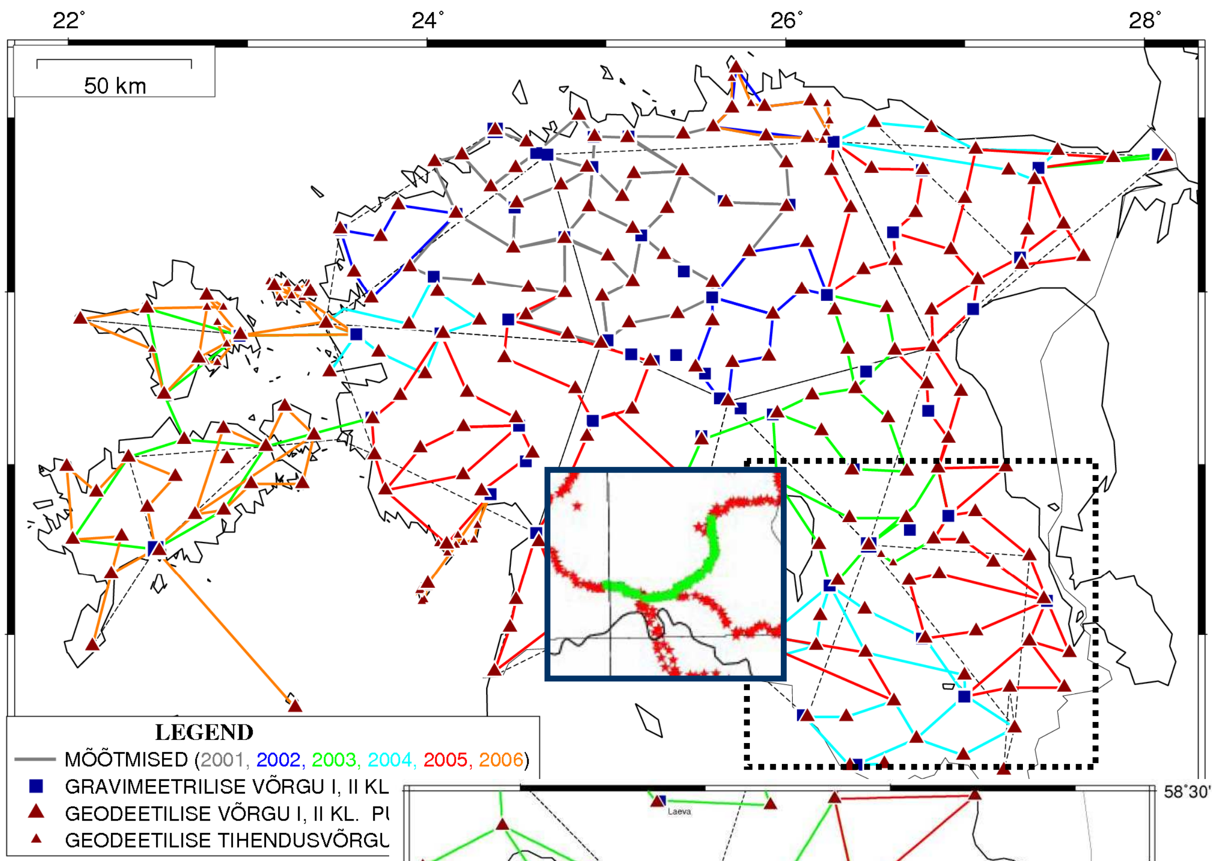


GV-EST95 – Unsolved issues

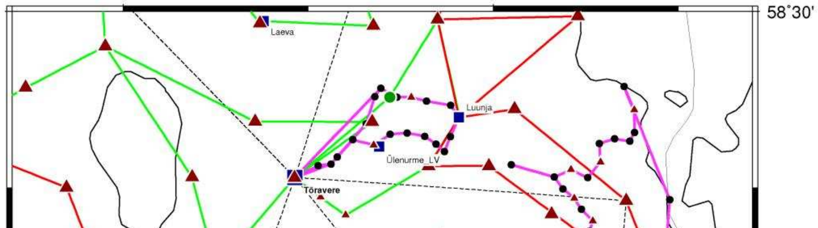
- Test and calibration of Scintrex CG5's
- Inclusion of temporal gravity changes:
  - PGR: based on the NKG2005LU?
  - the effect of water mass distribution
- Epoch of the network (2008.?)
- Management of the observation data
- Reduction of absolute gravity values and nonlinear gradient problem (next talk)



GV-EST95 –



- Geodesists
  - EULS, TUT
- Geologists
  - GSE, IG UT
- Metrologists
  - AS Metrosert (Central Office of Metrology in Estonia)
- International community
  - EGM, EGGP, UEGN,…



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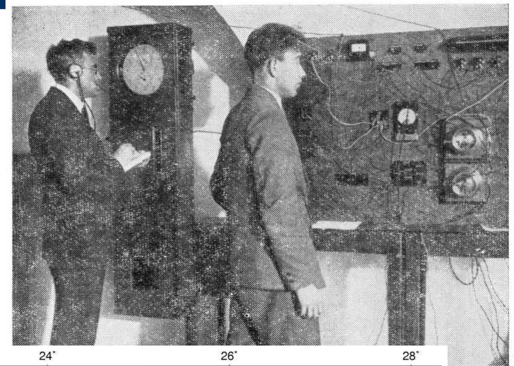
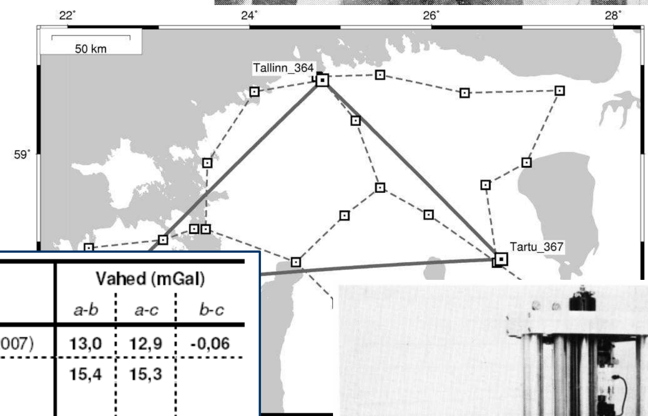
Estonian gravity network – Past

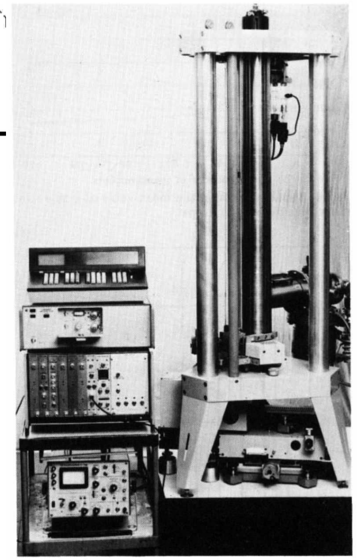
- Potsdami gravity system
  - BGK realization (1930-1955) **–12.9** mGal
  - SU realization(s) (1955-1975) **–15.4**
- I.G.S.N.71 **–13.9**
  - SU realization(s) (1975-1995) **–0.08**
- Estonian gravity system EGS since 1995
- Differences:

СЕКРЕТНО





Jknr	Punkt Nimi	nr	Potsdam a	IGNS71 b	EGS c	Vahed (mGal) a-b	a-c	b-c
1	Toompea	5703844	839,6 (1930)[1]	826,63 (1992)[6]	826,690 (2007)	13,0	12,9	-0,06
			842,0 (1957)[2]			15,4	15,3	
			-2,4					
2	Tartu LV *	367	796,73 (1955)[2]		781,680 (2002)		15,05	
			795,4 (1958)[3]				13,7	
			1,3					
3	Tallinna LV *	389	843,4 (1965)[3]	829,81 (1983)[5]	829,742 (1992/95)	13,6	13,7	0,07
					829,711 (2005)[5]		13,7	0,10
					0,031			
4	Harku	80017 **	843,93 (1966)[4]	828,61 (1992)[6]	828,534 (2003)	15,32	15,40	0,08
5	TTÜ	80910 **	848,64 (?)[5]	834,742 (1975)[6]	834,638 (2003)	13,90	13,99	0,104
				834,706 (1986)[5]		13,93	14,00	0,068
				0,036				
6	Tallinna LV	81691 **		830,466 (1986)[5]	830,306 (2005)			0,160
				830,384 (1993)[5]				0,078
				0,082				



Great challenge

- Complete solution will (or should) be presented at the end of 2009

VG is constant!

NOT!

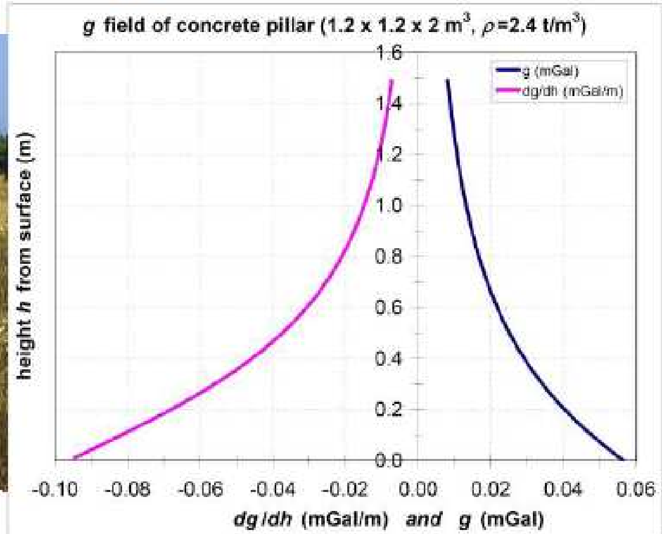
(B. Sagdiyev)

Last year at WG meeting...

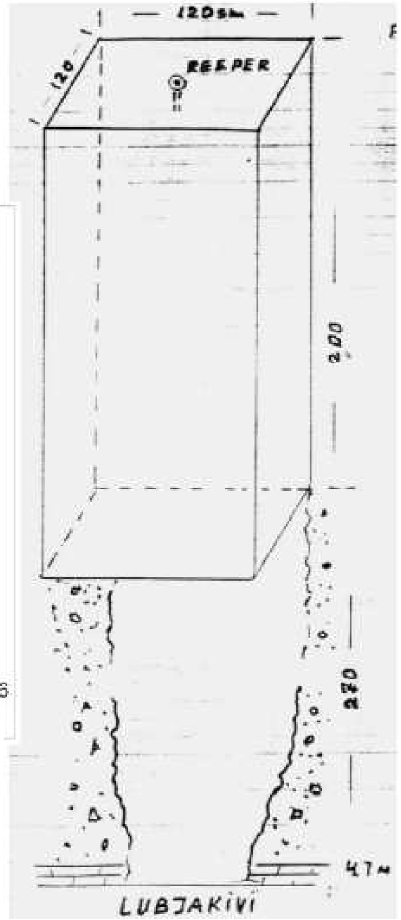
Vertical gravity gradient

- Is VGG constant?
- In Suurupi (probably) NO!

North-Estonian Klint (limestone cliff), 25…70 m high in Estonia, Suurupi pier 30 m away



Attraction of a single rectangular prism



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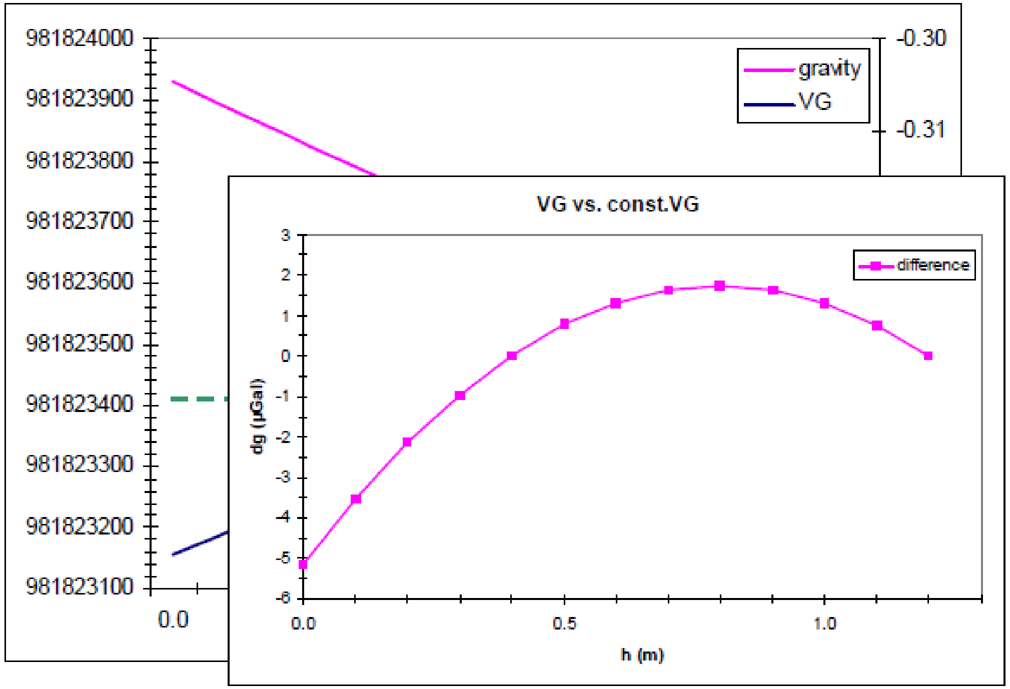
23. 04. 2008

Last year at WG meeting...

- Fitting the function (by WLSQ)

$$g(h)=\sum_{i=0}^{m} c_i h^i \approx g_0 + c_1 h + c_2 h^2 \Rightarrow g(h_2)-g(h_1)=c_1(h_2-h_1)+c_2(h_2^2-h_1^2)$$

	g0=	981823930.0	uGal
	c1=	0.3563	uGal/mm
FIT	c2=	-1.084E-05	
	g(h_i)=g_0 + c1*h_i + c2*(h_i)**2		
Weight P	dg(dh)=c1*(h2-h1)+c2*(h2^2-h1^2)		
		v	
0.44	263.86	1.8	ei ole sobitam
0.44	346.59	-3.5	
1.00	339.85	-2.0	
1.00	339.41	-0.6	
1.00	358.94	-1.0	
1.00	254.76	-0.2	
0.44	412.95	0.5	
1.00	253.73	1.4	
0.44	412.27	-0.4	
1.78	260.16	1.3	
1.00	391.57	0.0	
1.78	260.17	-2.1	
1.78	391.59	0.0	
1.78	246.11	0.9	
1.78	417.50	0.1	
1.78	255.49	0.9	
1.78	427.11	1.1	
1.78	255.81	0.3	
1.78	427.41	1.1	
1.00	981823518.0	0.0	
	LSQsum	RMS	
	30.01	1.29	



Results in Suurupi

23. 04. 2008

Proposal by Jaakko Mäkinen:

- remove the theoretical influence of local masses (massive pier)
- fit the constant VG or polynomial function
- restore the theoretically calculated attraction

so called “remove-restore” (rmr) method

Functional model

Observed gravity difference *dg* between heights h_1, h_2 and modeled attraction of the pier $M(h)$:

$$dg - \left(M\left(h_2\right) - M\left(h_1\right)\right) = \sum_{i=1}^{l} c_i \left(h_2^i - h_1^i\right)$$

Functional model

Modification: let pier's density *rho* be also unknown parameter*. Then...

observed gravity differences at h_1, h_2:

$$dg = \sum_{i=1}^{l} c_i (h_2^i - h_1^i) + rho * (M'(h_2) - M'(h_1))$$

Introduce also fixed (absolute) gravity at obs height:

$$g(h_{obs}) = g(h_0) + \sum_{i=1}^{l} c_i (h_{obs}^i - h_0^i) + rho * (M'(h_{obs}) - M'(h_0))$$

* Refers to Nettleton's method

M' – so called modelled pseudo-gravity

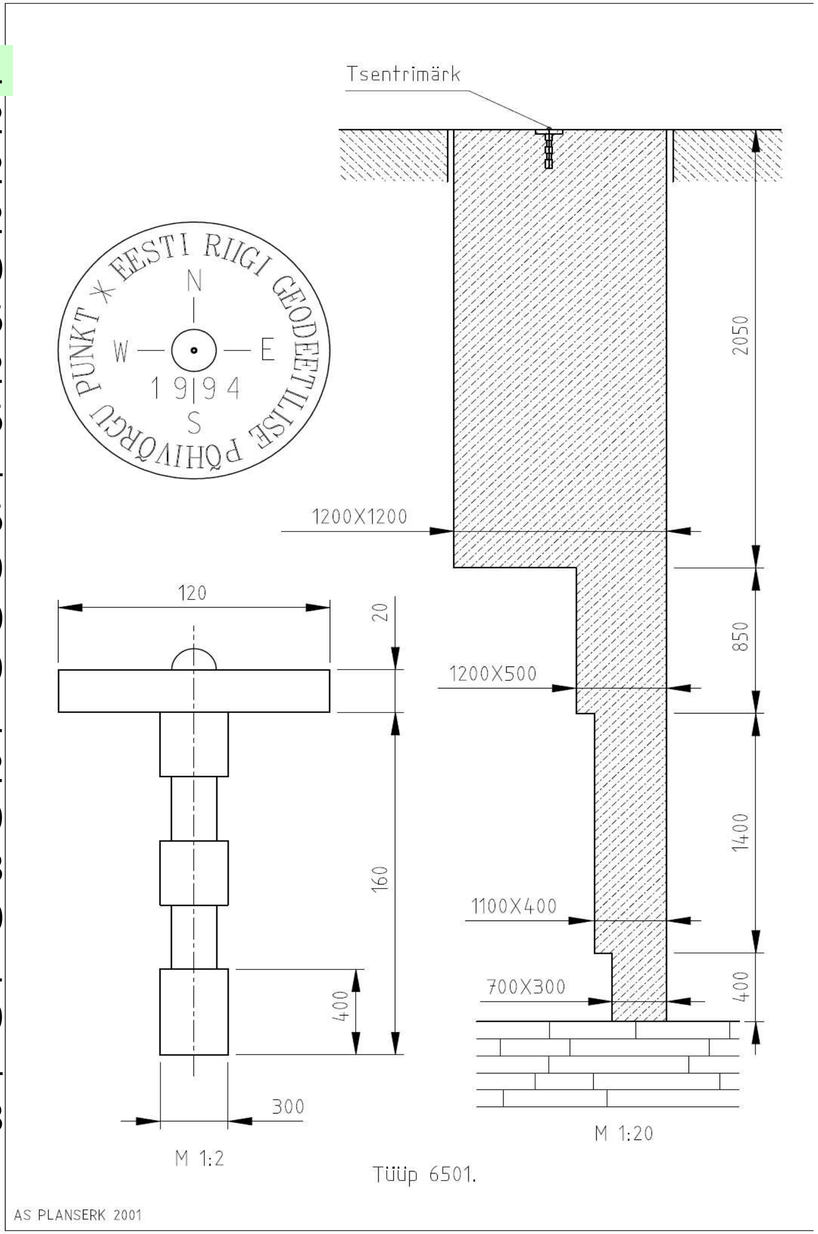
Test data (Suurupi):

Jknr	Asutus	Mõõtja	Gravimeeter	Kuupäev	Mõõtmiskõrgus		Sensori kõrgus		Kõrguskasv,määram		g juurdekasv,määram	
					h1	h2	sh1	sh2	dh	dh_err	dg	dg_err
							mm				µGal	
1	FGI	JM	LCR-G600	1995-09-30	201	938	42	779	737	1	258.7	1.0
2	FGI	JM	LCR-G600	1995-10-01	201	938	42	779	737	1	258.3	0.9
3	ELB	TO	CG5-36	2004-11-12	313	1323	102	1112	1010	2	341.3	1.0
4	ELB	TO	CG5-36	2004-12-17	317	1307	106	1096	990	2	337.9	0.4
5	ELB	TO	CG5-10092	2004-12-17	322	1311	111	1100	989	2	338.8	0.6
6	IfE	LT	CG3-4492	2007-07-09	427	1483	229	1285	1056	1	357.9	0.7
7	ELB	TO	CG5-36	2007-09-26	312	1048	101	837	736	2	254.6	0.6
8	ELB	TO	CG5-36	2007-09-26	312	1523	101	1312	1211	2	413.7	0.6
9	ELB	TO	CG5-10092	2007-09-26	313	1046	102	835	733	2	255.1	0.5
10	ELB	TO	CG5-10092	2007-09-26	313	1522	102	1311	1209	2	411.7	0.5
11	ELB	TO	CG5-36	2007-11-15	313	1065	102	854	752	2	261.1	0.4
12	ELB	TO	CG5-36	2007-11-15	313	1459	102	1248	1146	2	391.6	0.5
13	ELB	TO	CG5-10092	2007-11-15	312	1064	101	853	752	2	258.6	0.3
14	ELB	TO	CG5-10092	2007-11-15	312	1458	101	1247	1146	2	391.6	0.4
15	ELB	TO	CG5-10092	2008-02-05	314	1024	103	813	711	2	246.8	0.4
16	ELB	TO	CG5-10092	2008-02-05	314	1539	103	1328	1225	2	417.6	0.4
17	ELB	TO	CG5-36	2008-02-21	309	1047	98	836	738	2	256.2	0.4
18	ELB	TO	CG5-36	2008-02-21	309	1563	98	1352	1254	2	427.9	0.4
19	ELB	TO	CG5-10092	2008-02-21	310	1049	99	838	739	2	256.0	0.4
20	ELB	TO	CG5-10092	2008-02-21	310	1565	99	1354	1255	2	428.2	0.5
21	FGI	MBK	CG5-10052	2008-07-17			121	1288	1167	1	396.9	0.9
22	FGI	MBK	CG5-10052	2008-07-17			121	789	668	1	231.8	1.0

IfE abs.meas. in 2007

Test data (SUUR):

	g	h1	h2	M1	M2
1	981823518	0.0000	1.2000	0.02367	0.00451
2	-258.7	0.0420	0.7790	0.02206	0.00722
3	-258.3	0.0420	0.7790	0.02206	0.00722
5	-337.9	0.1060	1.0960	0.01979	0.00502
6	-338.8	0.1110	1.1000	0.01963	0.00500
7	-357.9	0.2290	1.2850	0.01611	0.00415
8	-254.6	0.1010	0.8370	0.01996	0.00672
9	-413.7	0.1010	1.3120	0.01996	0.00405
10	-255.1	0.1020	0.8350	0.01993	0.00674
11	-411.7	0.1020	1.3110	0.01993	0.00405
12	-261.1	0.1020	0.8540	0.01993	0.00659
13	-391.6	0.1020	1.2480	0.01993	0.00430
14	-258.6	0.1010	0.8530	0.01996	0.00659
15	-391.6	0.1010	1.2470	0.01996	0.00431
16	-246.8	0.1025	0.8130	0.01991	0.00692
17	-417.6	0.1025	1.3275	0.01991	0.00399
18	-256.2	0.0980	0.8360	0.02006	0.00673
19	-427.9	0.0980	1.3520	0.02006	0.00390
20	-256	0.0990	0.8380	0.02003	0.00671
21	-428.2	0.0990	1.3540	0.02003	0.00389
22	-396.9	0.1210	1.2880	0.01930	0.00414
23	-231.8	0.1210	0.7890	0.01930	0.00713

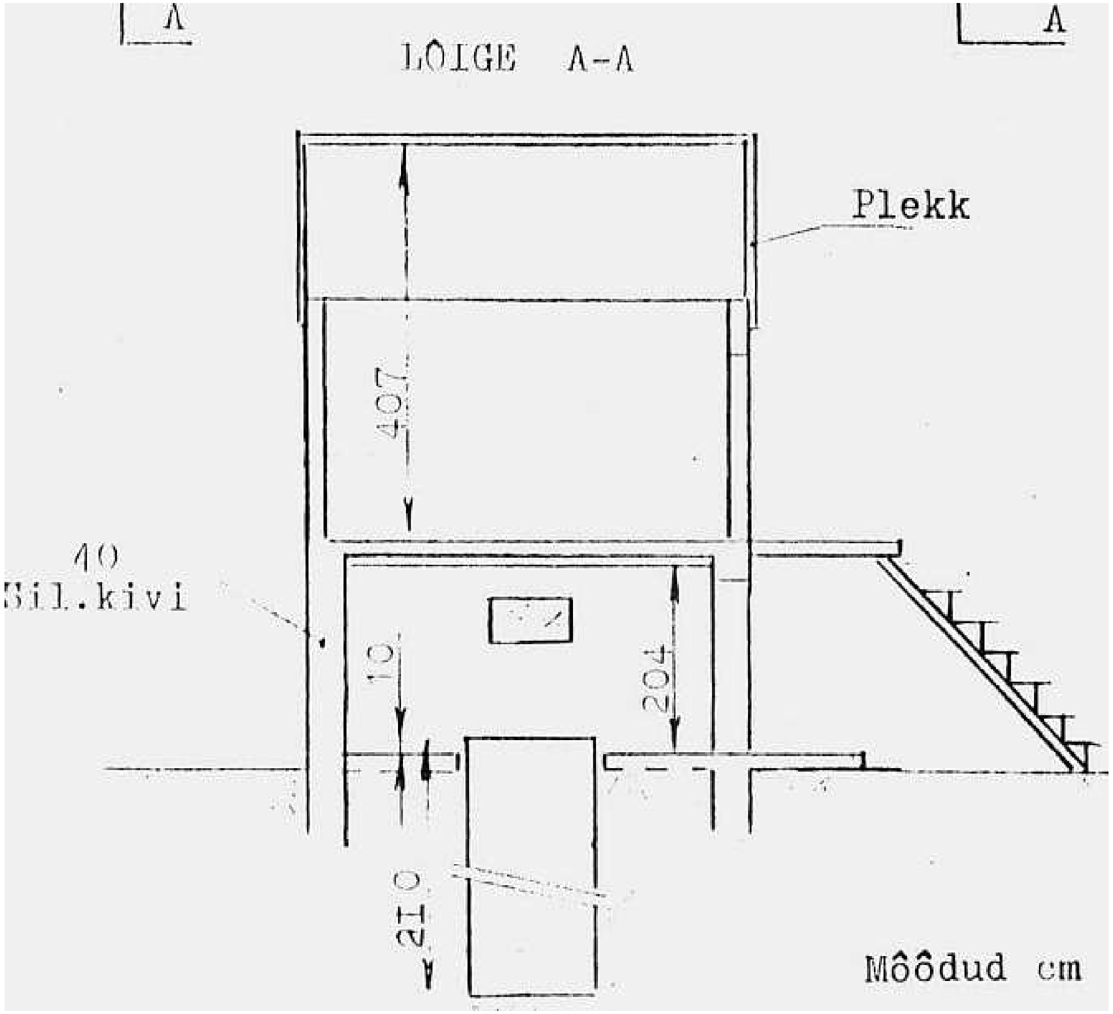


Results, source Wzz.f (SUUR)

- Constant VG: RMS=2.95 uGal, g(0)=981 823 929.3 uGal
- 2nd order polynomial: RMS=1.01, g(0)= ... 930.3
- Const VG + rmr (const. rho=2 t/m3): RMS=1.99, g(0)=... 932.5 (*M* = rectangular 1.2x1.2x2.05)
- 2nd ord.pol + rmr (const. rho=2 t/m3): RMS=0.70, g(0)=... 931.9
- Const VG + rmr (rho param.):RMS=0.79, g(0)=... 931.3 (adj. rho~1.22 t/m3)
- 2nd ord.pol + rmr (rho param.):RMS=0.67, g(0)=... 932.4 (adj. rho~2.71)

Test data (TORA):

dg	sh1	sh2
-239.8	0.040	0.780
-241.0	0.040	0.780
-326.4	0.236	1.266
-370.4	0.121	1.290
-213.5	0.121	0.791
-327.4	0.107	1.140
-328.7	0.110	1.145
-199.1	0.108	0.727
-349.6	0.108	1.219
-198.5	0.108	0.726
-352.2	0.108	1.219
-218.7	0.104	0.786
-376.8	0.104	1.291
-218.1	0.106	0.790
-376.6	0.106	1.294
981759285.0	0	1.2



Results (TORA):

- Wzz_constVG_rho-0.out: RMS=2.32 uGal,
  g(0)= 981 759 666.6 uGal
- Wzz_2nd_rho-0.out: RMS= 1.33,
  667.0
- Wzz_constVG_rho-fix.out: 3.14 (rho=2000 kg/m3),
  669.0
- Wzz_2nd_rho-fix.out: 1.16 (rho=2000 kg/m3),
  668.3
- Wzz_constVG_rho-adj.out: 1.25,
  667.6 (rho=809 ± 143)
- Wzz_2nd_rho-adj.out: 1.11,
  669.4 (rho=3699 ± 1610)

Results (TORA):

- Wzz_constVG_rho-0.out:

```
dof: 14
- solution OK, max loss of digits:  0.0
Solution:
981759285.0  0.000  1.200   981759285.0          0.0
     -239.8  0.040  0.780        -235.3          4.5
     -241.0  0.040  0.780        -235.3          5.7
     -326.4  0.236  1.266        -327.6         -1.2
     -370.4  0.121  1.290        -371.8         -1.4
     -213.5  0.121  0.791        -213.1          0.4
     -327.4  0.107  1.140        -328.5         -1.1
     -328.7  0.110  1.145        -329.1         -0.4
     -199.1  0.108  0.727        -196.9          2.2
     -349.6  0.108  1.219        -353.3         -3.7
     -198.5  0.108  0.726        -196.5          2.0
     -352.2  0.108  1.219        -353.3         -1.1
     -218.7  0.104  0.786        -216.9          1.8
     -376.8  0.104  1.291        -377.5         -0.7
     -218.1  0.106  0.790        -217.5          0.6
     -376.6  0.106  1.294        -377.8         -1.2
RMS=  2.32387847
SIGMA=  2.48433058
-----------------------
g(0) and its stdev=  981759667.  2.6173454
c( 1),stdev= -318.017912  0.686475769
rho=  0.
```

- Wzz_2nd_rho-adj.out:

```
dof: 12
- solution OK, max loss of digits:  3.8
Solution:
981759285.0  0.000  1.200   981759285.0          0.0
     -239.8  0.040  0.780        -239.2          0.6
     -241.0  0.040  0.780        -239.2          1.8
     -326.4  0.236  1.266        -323.8          2.6
     -370.4  0.121  1.290        -370.5         -0.1
     -213.5  0.121  0.791        -214.2         -0.7
     -327.4  0.107  1.140        -327.6         -0.2
     -328.7  0.110  1.145        -328.1          0.6
     -199.1  0.108  0.727        -199.0          0.1
     -349.6  0.108  1.219        -352.3         -2.7
     -198.5  0.108  0.726        -198.7         -0.2
     -352.2  0.108  1.219        -352.3         -0.1
     -218.7  0.104  0.786        -218.6          0.1
     -376.8  0.104  1.291        -376.7          0.1
     -218.1  0.106  0.790        -219.1         -1.0
     -376.6  0.106  1.294        -377.0         -0.4
RMS=  1.10869842
SIGMA=  1.28021466
-----------------------
g(0) and its stdev=  981759669.  1.71348266
c( 1),stdev= -214.996062  49.8682065
c( 2),stdev= -34.0247221  18.9000793
rho=  3698.54987
...and its stdev=  1610.44362
-----------------------
```

Conclusions:

- Results presented here are very preliminary (aim is to just present idea)
- RMS values show better performance with „rmr“
- Overfit danger: unrealistic results (*rho*)
- Absolute gravity value at BM level may differ several uGal and depends on how to model local gravity field
- Always test nonlinearity of VG.
  Avoid to use constant VG!